FLUCTUATIONS OF THE NUMBER OF SOLID-PHASE PARTICLES IN A FLUIDIZED BED

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An analytic expression is obtained for calculating the relative fluctuation of the number of particles in a fluidized bed. The fluctuation of the number of particles in a bed consisting of glass pellets fluidized with water has been experimentally investigated. The relative fluctuations obtained in these experiments are compared with the theoretical values for a fluidized bed and an ideal gas.

To solve problems of heat and mass transfer and mixing and separation of the solid phase in a fluidized bed it is necessary to know not only the average porosity of the bed but also details of its structure and the fluctuations of particle density.

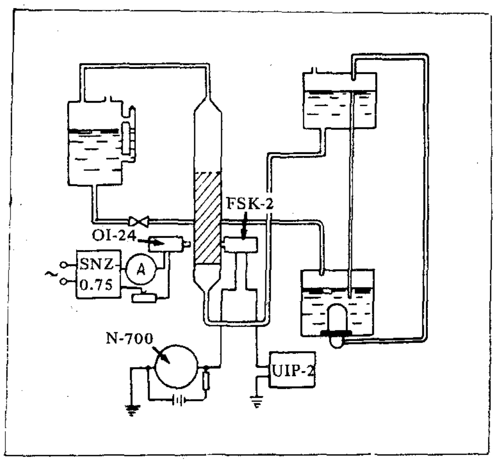

Fig. 1. Schematic of experimental apparatus.

In the general case, if the particles are randomly distributed, then, according to Smolukhovskii [2], the probability of a particle remaining within a measured volume V will be

$$p(n) = \frac{n_0^n}{n!} \exp[-n_0]. \tag{1}$$

In a fluidized bed the entire "probability mass" is distributed in the interval of particle numbers from 0 to n_1. Since the sum of the probabilities must form a complete group, passing from summation to integration, we can write

$$p(n) = \int_0^{n_1} \frac{n_0^n}{n!} \exp[-n_0]\, dn = 1. \tag{2}$$

Experiment [2] gives a normal fluctuation probability distribution for a fluidized bed. For a normal distribution the distribution function, with account for the "probability mass" interval, has the form [3]

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^{n_1} \exp\left[-\frac{(n-n_0)^2}{2\sigma^2}\right] dn = 1. \tag{3}$$

Differentiating (2) and (3) with respect to the upper limit and obtaining the joint solution of the resulting expressions for σ^2, we have

$$\sigma^2 = (n_1 - n_0)^2 \Big/ 2\left(n_0 + \ln n_1! - n_1 \ln n_0 - \frac{1}{2}\ln 2\pi - \ln\sigma\right). \tag{4}$$

According to Stirling's formula

$$\ln n_1! = \frac{1}{2}\ln 2\pi n_1 + n_1 \ln n_1 - n_1. \tag{5}$$

With account for (5), (4) assumes the form

$$\sigma^2 = (n_1 - n_0)^2 \Big/ 2\left(n_0 - n_1 + n_1 \ln\frac{n_1}{n_0} + \frac{1}{2}\ln n_1 - \ln\sigma\right). \tag{6}$$

Equation (6) can be used to calculate the variance of the number of particles in the fluidized bed. Since the relative fluctuation $\delta = \sigma/n_0$, (6) can be transformed to

$$\delta = \frac{n_1 - n_0}{1.41 n_0 \left[n_0 - n_1 + (n_1 + 1)\ln\frac{n_1}{n_0} - \frac{1}{2}\ln n_1 - \ln\delta\right]^{1/2}}. \tag{7}$$

Equation (7) can be used to calculate the relative fluctuation of the number of particles in the measured volume for all fluidization conditions characterized by a normal distribution.

The relation obtained was experimentally verified on the apparatus shown schematically in Fig. 1. A fluidized bed of glass pellets 0.001–0.002 m in diameter was created with water in a 0.04 × 0.045 m column made of plastic. Water was supplied to the distributor from a constant-level tank. The bed was traversed by a beam of light from a OI-24 illuminator. The light was received by a FSK-2 photocell, the circuit of which included the galvanometer of a N-700 oscillograph. The constant component of the electric signal supplied to the N-700 was partially chopped by inserting a battery with reverse polarity. The chopped part of the signal remained constant in magnitude in all the experiments and was equal to c = 0.108 m of deflection of the galvanometer beam on the oscillograph chart.

Thus, the variation of the number of pellets in the

Table

Results of Calculation of $\bar{\delta}$

H, mm	n_0	$\bar{L}$, mm	σ_e, mm	δ	$\bar{\delta}$
70	642	87.0 64.6 55.8 47.9 33.4 19.3	5.51 5.50 4.93 4.75 4.26 4.30	0.0282 0.0319 0.0300 0.0304 0.0301 0.0338	0.0307
89	505	73.4 59.9 55.0 44.6 38.9 18.5 17.9	7.94 6.64 6.74 7.76 5.65 5.66 5.60	0.0438 0.0395 0.0413 0.0500 0.0384 0.0450 0.0445	0.0432
98	458	71.6 24.6	8.80 5.96	0.0438 0.0450	0.0469
115	391	85.0 68.8 26.7	10.45 7.46 6.56	0.0542 0.0421 0.0485	0.0483

measured volume was converted into an electric signal recorded on the oscillograph, whose chart also displayed time marks every 0.5 sec.

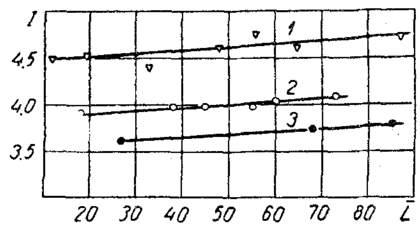

Fig. 2. Lamp current I (A) as a function of the mean deflection of the galvanometer beam on the oscillograph chart $\bar{L}$ (mm) for the bed heights of: 1) 0.070 m, 2) 0.089 m, and 3) 0.115 m.

The experiments were conducted at different water supply rates. The height of the bed, measured with a KM-6 cathetometer, varied from 0.03 m for the stationary bed to 0.115 m at the maximum rate of water supply. The illuminator lamp was supplied from a ferro-resonance stabilizer. A class-0.1 ammeter was used to check the constancy of the current.

Density fluctuations were recorded at different intensities of illumination (i.e., at different lamp currents) under the same hydrodynamic conditions. The oscillograms obtained were examined by the methods of mathematical statistics. On these oscillograms we measured the deviations of the curve from the base line every half-second. The values of these deviations were incorporated in distribution tables. On the basis of these distribution tables we calculated: the arithmetic mean deviation $\bar{L}$, the variance σ_e^2, and the standard deviation σ_e.

The dependence of L on the lamp current (Fig. 2) is linear. This suggests that the mean deviation and the fluctuations of the photocell current are linearly related with the number of particles in the measured volume.

Hence, the mean deflection of the galvanometer beam on the oscillograph chart $\bar{L}$ and the mean number of particles in the measured volume n_0 are related by the expression

$$n_0 = A(\bar{L} + c), \qquad (8)$$

while for the standard deviation of the number of particles we have

$$\sigma = A\sigma_e. \qquad (9)$$

In view of (3) and (9), the expression for calculating the relative fluctuation from the experimental data takes the form

$$\delta = \sigma_e/(\bar{L} + c) = \sigma/n_0. \qquad (10)$$

For the same hydrodynamic conditions (H = const) δ must be a constant and independent of the illumination of the bed. Substituting into (10) the values of L and σ_e obtained in the experiments for a single fluidization regime at different illuminations, we can calculate the mean values of δ from the experimental data (see the table).

To compare the experimental values of δ with those calculated from (7), it is necessary to have data on the mean number of particles in the measured volume corresponding to the found values of δ.

In the first approximation we may assume that the particles are distributed uniformly over the entire volume of the bed (ideal fluidized bed). The mean number of particles for the ideal bed

$$n_0 = \frac{N}{sH} V, \qquad (11)$$

where V is taken as the volume of the cylinder cut out of the bed by the photocell aperture.

By substituting into (11) the height of the compact bed H = 0.03 m we obtained the value n_1 = 1506, the number of particles in the measured volume for a compact bed.

Values of n_0 calculated from (11) and the corresponding δ are presented in the table and Fig. 3. The figure also shows the dependence of the relative fluctuation δ on the mean number of particles in the measured volume computed from (7) for n_1 = 1506.

As may be seen, the experimental points are grouped closely about the experimental curve. The broken line shows the relative fluctuation of the number of particles in an ideal gas calculated from the equation $\delta = 1/\sqrt{n_0}$.

Clearly, the course of the curves is quite different and only on a certain small interval of values of n_0 do the fluctuations approach each other.

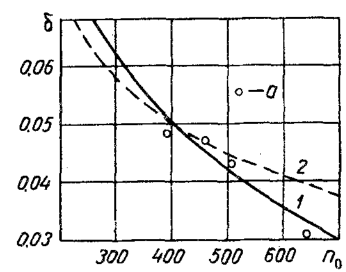

Fig. 3. Variation of relative fluctuation as a function of mean number of particles: 1) calculated for a fluidized bed, 2) for an ideal gas, 3) experimental points.

NOTATION

n—number of particles in measured volume, n_0—mean number of particles, n_1—number of particles in measured volume for compact bed, p(n)—probability of remaining in measured volume, σ—standard deviation of number of particles, $\bar{L}$—mean deviation of galvanometer beam on oscillograph chart, H—height of bed, N—number of particles in bed, s—cross-sectional area of free column, σ_e—standard deviation of galvanometer beam on oscillograph chart, δ—relative fluctuation.

REFERENCES

1. A. K. Timiryazev, Kinetic Theory of Matter [in Russian], Uchpedgiz, Moscow, p. 89, 1956.
2. V. I. Kislykh, Khim. prom-st, no. 6, 1965.
3. V. G. Levich, Introduction to Statistical Physics [in Russian], GITTL, p. 289, 1954.
4. Encyclopedic Machine-Building Handbook [in Russian], vol. 1, part 1, GITTL, Moscow, 1947.

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